New Relationships Between Certain Physico-chemical Constants Established on the Dimensional Analysis

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This study proposes a method based on dimensional analysis for establishing new relationships between certain fundamental physico-chemical constants. In accordance with this method every fundamental physico-chemical constant could be expressed by a "characteristic length". If a characteristic length associated with a physico-chemical constant is related to the characteristic length associated with the speed of light then the proper normalized value for every constant could be calculated. These normalized values are universal constants independent from any measure units systems. The relationships between these normalized values can be associated to geometrical ratios by using the so-called "characteristic length" similarly to the relationships existing between quadratic, rectangular and circular shapes. These relationships might offer another perspective for interpreting similar physico-chemical phenomena associated to atomic structure of matter.

Keywords: physico-chemical constants, dimensional analysis, characteristic length, similarity, golden number φ (phi)

The goal of this article is to introduce certain relationships between physico-chemical constants based on a dimensional analysis. These relationships might offer new and nonconventional tools for the study of atomic and subatomic structure of substances related to well-established physico-chemical principles. Concrete results of a such study based on dimensional analysis are presented in the article.

At the core of dimensional analysis is the concept of similarity. In physical terms, similarity refers to an equivalence between two things or phenomena that are actually different. The basic principle of dimensional analysis was established by Isaac Newton (1686) who referred to it as the "Great Principle of Similitude" [1]. Since then the literature has grown prodigiously in this field. As presented by Sonin [2] "Applications now include: aerodynamics, hydraulics, ship design, propulsion, heat and mass transfer, combustion, mechanics of elastic and plastic structures, fluid-structure interactions, electromagnetic theory, radiation, astrophysics, underwater and underground explosions, nuclear blasts, impact dynamics, chemical reactions and processing and also biology and even economics". The interest of the author of this article in dimensional analysis has resulted in a number of papers [3, 4].

The dimensional analysis is based on the relations between dimensionless groups. These relations have a general form as follows:

$$Q_1 = K\, Q_2^{n1}\, Q_3^{n2} \ldots Q_n^{nn} \qquad (1)$$

where:

K is a dimensionless constant;

Qs would include all the variables known to enter into the particular phenomenon;

Q1 might considered to be the quantity of principal interest,

n1, n2, … nn - numerical coefficients dependent on experimental data. It is generally accepted that dimensional analysis does not give quantitative information so that experiment must still be relied upon for that purpose [5]. The study presented in this article proves that some of the dimensionless criteria could be related to the quantitative information based on the numerical values of the fundamental physico- chemical constants, due to similarity of many physico-chemical phenomena which govern the natural processes.

Experimental part

Methods and results

The methods used in this article were presented by the author during the 14th National Conference on Physics [6], and so only a number of general elements are presented here. The method is based on the following analogy:

- each fundamental physico-chemical constant could be related to a "*characteristic length*" - x_{ch}; such a characteristic length could be associated with an existing one-dimensional physical entity;
- each fundamental physico-chemical constant may be expressed by means of the characteristic length x_{ch} (space), and a numerical value including some variables (mass, gravity, temperature, electricity, magnetism etc.);
- the numerical value which incorporates the variables referring to mass, gravity, temperature, electricity, magnetism etc., is considered to be a power of the number *2*; in this way, by means of the powers of *2*, any relationship regarding symmetry, dualism, partnership, existing between such variables can be expressed; it is superfluous to explain why *2* was chosen to express the symmetry, dualism or partnership, however it is worth mentioning that the idea behind this study came after the lecture of a number of books on the subject [7-10];
- a characteristic length x_{ch} is of an order of magnitude in the range of the Planck length ($l_p = (h G_k / c^3)^{1/2}$ and l_p is of an order of 10^{-35} *m* that is considered a reference length *lr*.

Using the above presented analogy every fundamental physico-chemical constant was expressed

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as the following relation:

$$q_c = x_{ch}^{\;n1} 2^{\,n2} \qquad (2)$$

where:

qc is the value of the constant;

x_{ch} - a characteristic length;

$n1$, and $n2$ - integer exponents. In this context the speed of light c or Plank constant h have been written as:

$$c=2.99792458\cdot 10^{8}\, m\cdot s^{-1}=(1.344315875\cdot 10^{-35}\, m)\cdot (2^{144}\, s^{-1}),$$
$$h=6.62606957(29)\cdot 10^{-34} Js=6.62606957(29)\cdot 10^{-34} m^2\cdot kg\cdot s^{-1}=$$
$$=(1.11634455(97)\cdot 10^{-35} m)^2\cdot(2^{122} kg\cdot s^{-1});$$

in this relation $J = m^2 . kg . s^{-2}$.

The results concerning the value of the fundamental physico-chemical constants (expressed in SI units) related to the characteristic lengths and powers of number *2* are presented in table 1. The numerical values of the fundamental constants in table 1 are in accordance with [11].

If a characteristic length of a fundamental constant x_{ch} is related to the characteristic length of the speed of light x_c certain normalized values $\overline{x}_{ch}$ are obtained (table 1 column 5). At first glance it appears that the normalized values in table 1 column 5 depend on the meter used as a standard unit for length. Calculus show that the values of the fundamental constants could be expressed using other length standard units that are different from the meter (inch, foot, yard, mile, astronomic unit etc.), but in the table 1 column 5 results are the same.

The values of the fundamental physico-chemical constants in table 1 column 4 have been calculated taking into account the Planck length l_p as a reference length l_r having a magnitude order of $10^{-35} m$. But calculations show that any other reference length lr could be considered. In all cases the results similar with $\overline{x}_{ch}$ in table 1 column 5 are obtained.

As a consequence, by taking the speed of light $\overline{x}_c$ as a reference (i.e. normalized value *1*) the normalized values $\overline{x}_{ch}$ of the fundamental physico-chemical constants can be expressed irrespectively of the measurement system. These normalized values do not depend on the measurement system used by the original constants.

The normalized values discussed above were calculated starting from the fundamental constants expressed in the SI units system. In SI it is well known that the elementary electric charge e and Faraday constant F are expressed in C (Coulomb). In this case the method presented above cannot be used. Whereas the normalized value for a fundamental physical constant does not depend on the units system, the electric charge e and Faraday constant F have been expressed using the CGS System. In the CGS System the electric charge e and Faraday constant F have the following values:

$$e=4.80320419\cdot 10^{-10} cm^{3/2}\cdot g^{1/2}\cdot s^{-1}=$$
$$=(1.248421053\cdot 10^{-33} cm)^{3/2}\cdot(2^{133}\cdot g^{1/2}\cdot s^{-1}); \qquad (3)$$
$$F=2.8925574\cdot 10^{14} cm^{3/2}\cdot g^{1/2}\cdot s^{-1} mol^{-1}=$$
$$=(1.24532280\cdot 10^{-33} cm)^{3/2}\cdot(2^{212}\cdot g^{1/2}\cdot s^{-1} mol^{-1}). \qquad (4)$$

In (3) and (4) e and F are expressed using characteristic lengths and the values have been obtained for a length of Planck $l_p = 1.61699 . 10^{-35}\, m = 1.61699 . 10^{-33} cm$. Keeping in mind these values, the characteristic length for the speed of light is $1.344325875. 10^{-33}$ *cm.* In these circumstances the normalized values for e and F are: $\overline{x}_e = 0.928666451$ and $\overline{x}_F = 0.926361745$ respectively. As in the previous cases the normalized values for e and F are the same when the normalized value for the speed of light x_c is equal to *1*.

In table 1 the normalized values were calculated to be very close to the normalized value for the speed of light $\overline{x}_c$

Table 1
NORMALIZED VALUES OF SOME FUNDAMENTAL PHYSICO-CHEMICAL CONSTANTS CALCULATED BY USING SO-CALLED "CHARACTERISTIC LENGTHS"

Unit of length – meter [m]; $l_P = 1.616199 \cdot 10^{-35}$ m				
	Values of the constants	*Units*	*Constants expressed using characteristic length - x_{ch} and power of 2* (see rel. 2)	*Normalized values* $\overline{x}_{ch} = x_{ch}/x_c$
1	*2*	*3*	*4*	*5*
c	$2.99792458\cdot 10^{8}$	$m\, s^{-1}$	$(1.344315875\cdot 10^{-35} m)\cdot(2^{144} s^{-1})$	$\overline{x}_c=1.000000000$
h	$6.626069572\cdot 10^{-34}$	$Js= m^2 kg\, s^{-1}$	$(1.11634455(97)\cdot 10^{-35} m)^2\cdot(2^{122}\, kg\, s^{-1})$	$\overline{x}_h=0.830418341$
$\hbar$	$1.054571726\cdot 10^{-34}$	$Js= m^2 kg\, s^{-1}$	$(1.25965994(49)\cdot 10^{-35} m)^2\cdot(2^{119}\, kg\, s^{-1})$	$\overline{x}_{\hbar}= 0.937026756$
$\hbar$*	$1.054571726\cdot 10^{-34}$	$Js\, m^{-1}= m\, kg\, s^{-1}$	$(1.318214657(50)\cdot 10^{-35} m)\cdot(2^{3}\, kg\, s^{-1})$	$\overline{x}_{\hbar^*}= 0.980584014$
G_k	$6.67384(80)\cdot 10^{-11}$	$m^3 kg^{-1}\, s^{-2}$	$(1.25985(11)\cdot 10^{-35} m)^3\cdot(2^{314}\, kg^{-1}\, s^{-2})$	$\overline{x}_{Gk}=0.937168587$
pV_m	$2271.0953(21)$	$Jmol^{-1}= m^2 kg\, s^{-2}\cdot mol^{-1}$	$(1.2675736(66)\cdot 10^{-35} m)^2\cdot(2^{243}\, kg\, s^{-2}\, mol^{-1})$	$x_{pVm}=0.942913558$
R	$8.3144620(94)$	$Jmol^{-1}K^{-1}= m^2 kg\, s^{-2}\cdot mol^{-1}K^{-1}$	$(1.2271356(92)\cdot 10^{-35} m)^2\cdot(2^{235}\, kg\, s^{-2}\, mol^{-1}K^{-1})$	$\overline{x}_R=0.912832851$
R_∞	10 973 731.568 539	m^{-1}	$(0.920\,144\,576\,50641)\cdot 10^{-35}\, m)^{-1}\cdot 2^{-93}$	$\overline{x}_{R\infty}=0.684470513$
F	$2.8925574\cdot 10^{14}$	$cm^{3/2}\cdot g^{1/2}\cdot s^{-1}\, mol^{-1}$	$(1.24532280\cdot 10^{-33} cm)^{3/2}\cdot(2^{212}\cdot g^{1/2}\cdot s^{-1} mol^{-1})$.	$\overline{x}_F=0.926361745$
e	$4.80320419\cdot 10^{-10}$	$cm^{3/2} g^{1/2}\cdot s^{-1}$	$(1.248421053\cdot 10^{-33} cm)^{3/2}\cdot(2^{133} g^{1/2}\cdot s^{-1})$;	$x_e= 0.928666452$

where: c is speed of light; e- electron charge; F- Faraday constant; G_k -constant of gravitation; h, ħ -Planck constant; ħ- Planck constant where 2π has dimension of a length; pV_m =RT – pressure x volume product constant from the ideal gas law of Clapeyron; R- universal gases constant; R_∞ -Rydberg constant; V_m = molar volume; $J=m^2 kg\, s^{-2}$ (Joule).*

equal to 1 but smaller than 1. At the same time normalized values larger than $\overline{x_c}=1$ could be calculated. The difference between smaller values and larger values is directly related to the power of the number *2*. For instance normalized values for pV_m and V_m are $\overline{x}_{pVm}$ = *0.9429135(58)* and $\overline{x}_{Vm}$*=0.8015095(46)* respectively. For these two cases the normalized values larger than *1* are:

$$\overline{x}_{pVm}=2^{1/2}\cdot 0.9429135(58)=1.3334811(41); \qquad (5)$$

$$\overline{x}_{Vm}=2^{1/3}\cdot 0.8015095(46)=1.0098387(48). \qquad (6)$$

The values smaller than $\overline{x_c}$ or larger than $\overline{x_c}$ are taken into account to explain more subtle relationships between normalized values (see above).

Results and discussions

It has been shown that the normalized values $\overline{x_{ch}}$ are physico-chemical fundamental constants and they do not depend by any measurement system. Being fundamental constants and reflecting in fact relationships between characteristic lengths (geometrical dimensions) it is logic to suppose that between these constants must exist relationships based on number *1*, number $1.25^{0.5,}$ number π or any other fundamental mathematical constant reflecting relationships between geometrical dimensions (quadratic, rectangular and circular shapes). A such fundamental mathematical constant that provides a relationship between quadratic, rectangular and circular dimensions is number φ (golden number phi) that is root of the equation $x^2 - x = 1$. The number φ is very important to be taken into account since the normalized value for the speed of light is $\overline{x_c}=1$.

Relationships between normalized values from table 1, number π, number φ and powers of number *2* are presented in table 2.

It is an evidence that relationships between normalized values in table 2 are not equal to number π or number φ they are only close to these mathematical numbers. It means that between normalized values there are not simple relationships but more complex connections and interdependence (see below). In these relationships the powers of the number *2* are the key element. At the same time there is a special relationship with the number *1* (direct related to number φ*)* that is added or subtracted in a direct connection with a certain state of the system under consideration. This complex aspect is shown below when the relationship between the speed of light *c* and the Planck constant is taken into consideration.

Relationships between speed of light and Planck constant

The relationships between speed of light and Planck constant are very well known and they can be found in any physico-chemicals technical book. Planck constant (denoted *h*, also called Planck's constant) describes the relationship between energy and frequency, commonly known as the Planck relation:

$$E_0=h\nu$$

The dimension of the Planck constant is equal to *energy* x *time* corresponding to the dimension of an action. It is an evidence that product between *h* and ν (having dimension time^{-1}) respectively $h\nu=E_0$ represents an energy. E_0 is in fact a minimum radiant energy - a "quantum". Since the frequency ν, wavelength λ, speed of light *c*, are related by $c=\nu\lambda$, the Planck relation for a photon can be expressed as:

$$E=hc/\lambda..$$

The Planck constant numerical value is equal to $6.62606957(29)\ .\ 10^{-34}$ *Js* [11].

For the purposes of this work the dimension of energy *J* is expressed in SI fundamental units respectively $m^2\ kg\ s^{-2}$. In this context *h* is expressed in $m^2\ kg\ s^{-1}$ (table 1).

In applications where circular movement is taken into consideration the Planck constant usually is related to 2π. The resulting constant is called the reduced Planck constant or Dirac constant. It is equal to the Planck constant divided by 2π and it is denoted ħ (or *"h bar"*, as it is often called*)*.

In this case numerical value of ħ is equal to $1.054571726(47)\cdot 10^{-34}$ *Js* [11]. Similarly with *h* the SI fundamental units for ħ are $m^2\ kg\ s^{-1}$.

Taking into account the numerical values for *h* and ħ the characteristic lengths and normalized values for these constants were calculated and presented in table 1, columns 4 and 5. At the same time in the table 2 (row 1 and 2) are presented the difference between the normalized values for these constants and normalized value for speed of light. The calculus show that these differences could be related to numbers π, φ or *(*φ*-1)* and power of number *2*. It has been shown above that since in these relationships the numbers π, φ or *(*φ*-1)* have not exact values between normalized values must exist more complex and more subtle connections.

This affirmation is supported by the following argument: if in the ħ definition formula the number 2π is not considered a dimensionless value but a circle having the length equal to 2π *[m]* (or radius equal to *1* m) then the Planck constant could be written as follows:

$$\hbar^*=h/2\pi[m]=1.054571726\cdot 10^{-34} m\cdot s^{-1}\cdot kg= \\ =(1.318214657(50)\cdot 10^{-35} m)\cdot(2^3 s^{-1} kg). \qquad (7)$$

In relation (7) Planck constant ħ* has dimensions of an impulse, $ms^{-1}\cdot kg$.

For this case the normalized value for the Planck constant $\overline{x\hbar^*}$ is equal to:

$$\overline{x\hbar^*}=1.318214657(50)\cdot 10^{-35} m/1.344315875237\cdot 10^{-35} m= \\ =0.980584014(35). \qquad (8)$$

Since the normalized value for the speed of light is $\overline{x_c}=1.00000$ the difference between the normalized value for the speed of light x_c and the normalized value for the Planck constant $\overline{x\hbar^*}$= *0.980584014*(35) is equal to $\overline{x_c}$- $\overline{x\hbar^*}$ = *0.019415985(65)* (table 2 row 3). If this value is divided by π is obtained:

$$0.019415985(65)/\pi=0.006180300(18), \qquad (9)$$

a result very close to $(\varphi-1)\cdot 10^{-2}$ where φ is root of the equation $x^2-x = 1$.

*S*ince speed of light *c* is expressed in $m\cdot s^{-1}$ and Planck constant ħ* in $m\cdot s^{-1}\cdot kg$ the difference between normalized values $\overline{x_c}$ - $\overline{x\hbar^*}$ *=0.019415985(65)* equal to $\pi 0.6180300(18)\ .\ 10^{-2}$ reflects only the mass influence. Having in view this aspect it is considered that a value equal to $\pi 0.6180300(18)$ very close to $\pi\cdot(\varphi-1)$ could be associated with the normalized value of the mass of an elementary physical entity having a circular shape. In this context *0.6180300(18)* corresponds to the diameter and $\pi\cdot 0.6180300(18)$ to the circumference. This elementary physical entity is considered a key element for

	Constants	- Differences between normalized values -Relationships of the normalized values with powers of number *2*	Power of number *2*	Comments
1	*c , h*	$\overline{x}_c - \overline{x}_h = 1.000000 - 0.8304183(41) = 0.1695816(58)$; $0.1695816(58) \cdot 2^{260} = \underline{\mathbf{3.1417}}943(20) \cdot 10^{77}$	*260*	*result close to π*
2	*c , ħ*	$\overline{x}_c - \overline{x}_ħ = 1.000000 - 0.93702675(60) = 0.06297324(41)$ $0.06297324(41)/2^{299} = \underline{\mathbf{6.1828}}3081(21) \cdot 10^{-92}$	*299*	*result close to (φ-1)*
3	*c , ħ**	$\overline{x}_c - \overline{x}_{ħ*} = 1.000000 - 0.980584014(35) = 0.019415985(65)$; $0.019415985(65)/\pi = \underline{\mathbf{6.180300}}(18)/2$	*1*	*result close to (φ-1)*
4	*c, G_K*	$\overline{x}_c - \overline{x}_{Gk} = 1.000000 - 0.93716(8587) = 0.06283(14)$; $0.06283(14) = 2 \cdot \underline{\mathbf{3.1415}}(70) \cdot 10^{-2}$	*1*	*result close to 2π· 10⁻²*
5	*c, pV_m*	$\overline{x}_{pVm} - \overline{x}_c = 1.3334811(41) - 1.000000 = 0.3334811(41)$; $0.333481141 / 2^{11} = (0.1 + 2 \cdot \underline{\mathbf{3.14162}}(94) \cdot 10^{-2}) \cdot 10^{-3}$	*11*	*result close to $0.1+2\pi \cdot 10^{-2}$*
6	*pV_m, R*	$\overline{x}_{pVm} - \overline{x}_R = 0.9429135(58) - 0.9128328(51) = 0.0300807(07)$; $\overline{x}_K = 0.0300807(07) \cdot 2^{207} = \underline{\mathbf{6.18724}}5(55) \cdot 10^{60}$	*207*	*result close to (φ-1)*
7	*c, e*	$\overline{x}_c - \overline{x}_e = 1.000000 - 0.9286664(52) = 0.0713335(48)$; $0.0713335(48)/2^{60} = \underline{\mathbf{6.187199}}(01) \cdot 10^{-20}$	*60*	*result close to (φ-1)*
8	*c, K*	$\overline{x}_c - \overline{x}_K = 1.000000 - 0.0300807(07) = 0.9699192(93)$; $0.9699192(93)/2^{190} = \underline{\mathbf{6.180682}}(32) \cdot 10^{-58}$	*190*	*result close to (φ-1))*

K refers to Kelvin grade; normalized value for K is equal to $\overline{x}_{pVm} - \overline{x}_R = \overline{x}_K = 0.0300807(07)$ (see row 6)

Table 2
RELATIONSHIPS BETWEEN NORMALIZED VALUES OF SOME FUNDAMENTAL PHYSICO-CHEMICAL CONSTANTS

explanation the mass-space relationship in the subatomic structure (see below).

Above presented affirmation is strengthened by the following arguments:

In the table 1 length is expressed in meters and time in seconds. In accordance with The General Conference on Weights and Measures meter and second are defined as follows: meter " is length of the path travelled by light in a vacuum in 1/299 792 458 of a second (17th CGPM)" [12]; second "is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom (Cs133)"(13th CGPM 1967/68) [13].

Having in view these definitions the second is a fundamental physical constant (transition between two hyperfine levels of the Cs133) and the meter is direct related to the second *via* speed of light. In this context *1 m/s* is a physical constant direct related to the dimension of a fundamental physical reality (transition between two levels). If the speed of light equal to *299 792 458 m/s* is related to *1 m/s* is obtained a normalized value equal to *299 792 458* for the speed of light.

Taking into account previously mentioned supposition that between time and space can be establish a relationship based on powers of number *2* then the speed of light could be expressed by the normalized value *299 792 458* multiplied or divided by powers of number *2*. If the normalized value *299 792 458* is divided by 2^{260} is obtained a result equal to $1.61816137427... \cdot 10^{-70}$ close to number φ divided by 10^{70}. It must be underlined that the power of number *2* equal to *260* is identical with the power of number *2* in relationship between *c* and *h* presented in table 2, row 1.

In accordance with rel. (7) the Planck constant could be written as $ħ^* = 1.054571726 \cdot 10^{-34} m \cdot s^{-1} \cdot kg$. Similarly to the normalized value for speed of ligth if $ħ^* = 1.054571726 \cdot 10^{-34} m \cdot s^{-1} \cdot kg$ is related to $1\ m \cdot s^{-1} \cdot kg$ is obtained a normalized value equal to $1.054571726 \cdot 10^{-34}$. If this normalized value is divided by 2^{119} is obtained $1.586743176 \cdot 10^{-70}$ a value close to $1.61816137427 \cdot 10^{-70}$ for speed of light both values being close to the number φ divided by 10^{70}. Taken into consideration that between speed of ligth *c* (expreesed in $m \cdot s^{-1}$) and Planck constant ħ* (expressed in $m \cdot s^{-1} \cdot kg$) there is a difference depending on only the mass it means that the difference between $1.61816137427 \cdot 10^{-70}$ and $1.586743176 \cdot 10^{-70}$ reflects the mass influence only. Calculus show that:

$$1.61816137427... \cdot 10^{-70} - 1.586743176... \cdot 10^{-70} = = 3.14181981779... \cdot 10^{-72} \quad (10)$$

The result in (10) respectively $3.14181981779... \cdot 10^{-72}$ is equal to $1/2^{294} = 3.1418198177... \cdot 10^{-89}$ multiplied by 10^{17}. It means that:

a) if the normalized value for speed of light equal to $1.61816137427 \cdot 10^{-70}$ (a value close to number φ divided by 10^{-70}) is multiplied by 2^{294} is obtained a normalized value for speed of light equal to $5.150395211... \cdot 10^{18}$; at the same time if the normalized value for Planck constant equal $1.586743176... \cdot 10^{70}$ is multiplied by 2^{294} is obtained a normalized value equal to $5.050395211... \cdot 10^{18}$; the difference between $5.150395211... \cdot 10^{-18}$ and $5.050395211... \cdot 10^{18}$ is equal to 10^{17} and this normalized value could be associated with the mass of an elementary physical entity having a linear shape;

b) when $\varphi \cdot 10^{-70}$ is taken into consideration the mass is related to 10^{17}; when instead of $\varphi \cdot 10^{-70}$ is taken into account $\varphi \cdot 10^{-87}$ the mass could be direct related to number *1*;

c) similarly with the previous case when the normalized value $\pi \cdot 0.6180300(18)$ close to $\pi \cdot (\varphi-1)$ has been associated with the mass of an elementary physical entity having a circular shape in this second case when the normalized value for speed of light is equal to $1.61816137427\ldots \cdot 10^{-70}$ (a value close to number φ divided by^{70}) the number *1* can be associated with a mass of an elementary physical entity having a linear shape.

As a consequence of the previous results we may say that: the normalized values for the mass could be obtained taking into account two elementary physical entities normalized values: one having a circular shape associated with number (φ -1) and second having a linear shape associated with number *1*. In both cases the relationships between normalized values are based on the powers of numbers *2* and *10*. It is more than evident that the sum of the elementary physical entities normalized values respectively $1 + (\varphi-1)$ is number φ .

It is presented above that normalized value for speed of light $1.61816137427\ldots \cdot 10^{-70}$ (close to number φ divided by 10^{70}) multiplied by 2^{294} is equal to $5.150395211\ldots \cdot 10^{18}$. Reciprocal of this value respectively $1/5.150395211\ldots \cdot 10^{18}$ is equal to $1.94159857462\ldots \cdot 10^{-19}$. It is very important to underline that $1.94159857462\ldots \cdot 10^{-19}$ is an exact calculated value and it is very close to the difference $\overline{x}_c - \overline{x}_{\hbar}{}^* = 0.019415985$ (table 2 row 3) divided by 10^{17}.

Having in view this exact calculated value for $\overline{x}_c - \overline{x\hbar}^* = 0.0194159857462\ldots$ a calculated value for Planck constant can be obtained in accordance with the following relationships (table 1 for $\overline{\hbar}$ and table 2 row 3 for $\overline{x}_c - \overline{x\hbar}^*$):

$$\overline{x_{\hbar}}^* = \overline{x_c} - 0.0194159857462 = 0.9805840142538 \quad (11)$$

$$\hbar = 0.9805840142538 \cdot 1.3443158752 \cdot 10^{-35} \cdot 2^3 = 1.054571725892 \cdot 10^{-34} \quad (12)$$

The Planck constant equal to $1.054571725892\ldots \cdot 10^{-34}$ is an exact calculated value obtained direct from the speed of light and it is very close to the experimental value equal to $1.054571726 \cdot 10^{-34}$. Taking into account relative standard uncertitude we may say that these two results are similar. If the future tests will prove that experimental value for Planck constant is different from the calculated value this difference is very important for establishing more subtle relationships existing between fundamental constants.

If the exact calculated value equal to *0.0194159857462…* is divided by π (9) is obtained:

$$0.0194159857462/\pi = 0.0061803002130195\ldots \quad (13)$$

This is an exact result close to $(\varphi-1) \cdot 10^{-2}$.

Taking into account this exact result for future argumentations it is considered as the normalized value equal to *0.61803002130195* (close to φ-*1*) corresponds to the diameter of an elementary physical entity and $\pi \cdot 0.61803002130195$ corresponds to its circumference.

The results discussed in this subchapter are presented in a schematic and unified way in figure1. This figure was achieved taking into account the core of the all relationships is number φ (golden number or golden section).

Having in view figure 1 it is easy to show that:

- the difference between speed of light *c* and number φ is equal to:

$$1.6181613742712\ldots - 1.61803398874989\ldots = $$
$$= 1.27385521400137\ldots \cdot 10^{-4} \quad (14)$$

- the elementary physical entity diameter is equal to:

$$1.941595857462\ \ldots / \pi = 0.6180300213019\ldots \quad (15)$$

- the difference between the elementary physical entity and (φ - 1) is:

$$0.618033988749\ldots - 0.6180300213019\ldots =$$
$$= 3.967447941636\ldots \cdot 10^{-6} \quad (16)$$

- the difference between elementary physical entity and speed of light is:

$$1.27385521400137\ldots \cdot 10^{-4} + 3.967447941636\ldots \cdot 10^{-6}/2 =$$
$$= 1.2936924537\ldots \cdot 10^{-4} \quad (17)$$

In (17) value $3.967447941636\ldots \cdot 10^{-6}$ is divided by *2* since it is taken into account radius of the elementary physical entity.

If the result in (17) is divided by 2^{156} is obtained:

$$1.2936924537\ldots \cdot 10^{-4} / 2^{156} = 1.416288475\ldots \cdot 10^{-47}, \quad (18)$$

a value very close to the decimal part of number π.

This normalized value equal to *0.1416288475…* close to the decimal part of number π is very important for the discussion in the next subchapter.

Relationships between other physyco-chemical constants

In the previous paragraph certain relationships between speed of light and Planck constant established on the base of the dimensional analysis are presented. It could arise legitimate questions as: May could somebody be interesting in such relationships? How could these relationships be used? In our opinion these relationships offer a nonconventional way for studying fundamental structure of matter. In this way certain examples are presented below:

1. When the relation between the speed of light and the Planck constant has been discussed it was shown the normalized value equal to *0.61803002130195…* very close to φ-*1*could be associated with the normalized value of a mass of an elementary physical entity having a circular shape. At the same time the normalized value equal to *1* could be associated with a mass of an elementary physical entity having a linear shape. We suppose there is a physical entity having a normalized value equal to *6.1803002130195…*(a value corresponding to *10* elementary physical entities) that is very close to *10(φ-1)* and this value is multiplied by *2* eight times and following each multiplication a normalized value equal to *1* is added. It is obtained finally:

$$6.1803002130195\ldots \cdot 2+1] \cdot 2+1] \cdot 2+1] \cdot 2+1] \cdot 2+1] \cdot 2+1] \cdot 2+1] \cdot 2+1] =$$
$$= 1836.156855\ldots \quad (19)$$

It is important to underline as *1836.156855* is a normalized value associated with a mass and it is very close to the experimental proton-electron mass ratio equal to *1836.1526724* [11]. Considering the result of rel. (19) it means that the proton mass could be obtained starting from *10* ten physical entities having its mass associated with a normalized value equal to

6.1803002130195… very close to *10(φ-1)*. Having in view the experimental proton-electron mass ratio equal to *1836.1526724* a normalized value for the *10* physical entities equal to *6.18028387656* could be calculated in accordance with (20):

$$1836.1526724 :2\text{-}1] :2\text{-}1]:2\text{-}1]:2\text{-}1] :2\text{-}1] :2\text{-}1]:2\text{-}1] :2\text{-}1] =6.18028387656 \quad (20)$$

If from *6.1803002130195…* is subtracted *6.1802838765625…* (see result in (20)) is obtained:

$$6.1803002130195...-.6.1802838765625...=1.6336457...\cdot 10^{-4} \quad (21)$$

If the result *1.6336457…* . 10^{-4} is divided by 2^{60} is obtained:

$$1.6336457...\cdot 10^{-4} / 2^{60}=1.41696177...\cdot 10^{-22} \quad (22)$$

a value close to *0.1416288475…* the result in (18) and close to the decimal part of number π.

2. Taking into account the normalized values a relationship between space ratios could be established as well. For this task it is taken into consideration the general relation on gasses, respectively:

$$pV_m = RT$$

It has been shown in rel. (5) that the normalized value for the pV_m constant is equal to $\overline{x}_{pV_m} = 1.3334811(41)$. If it is taken into account the difference between normalized value for the pV_m constant and the normalized value for the speed of light it is obtained:

$$\overline{x}_{pV_m} - \overline{x}_c = 1.3334811(41) - 1.000000 = 0.3334811(41)$$ *(see table 2 row 5)*

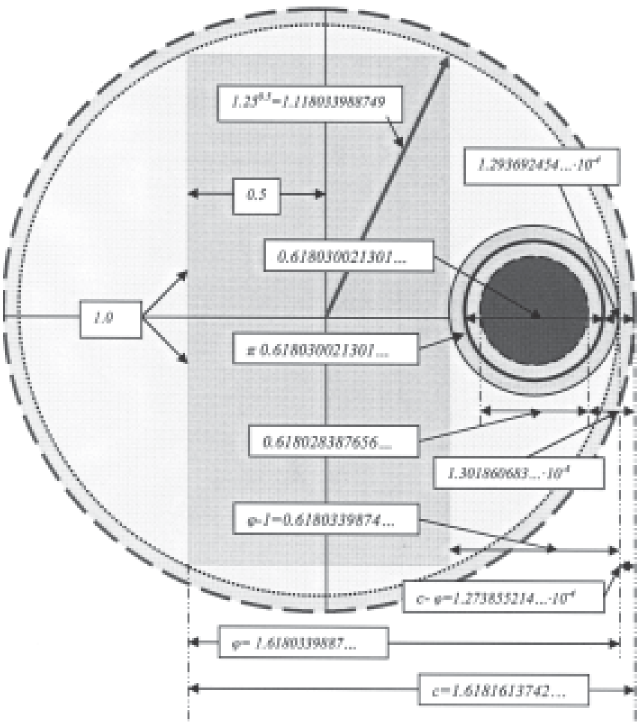

Fig. 1. Relationships between normalized values of some physico-chemical universal constants (values are schematic and presented no mathematic magnitude)

If this value is divided by 2^{12} is obtained:

$$0.333481141 / 2^{12} = 8.141629419 \,.\, 10^{-5} \quad (23)$$

a result very close to $(5+\pi)\cdot 10^{-5}$. At the same time decimal part of *8.141629419* respectively *0.141629419* is very close to the result in (18). It means that between elementary physical entity and speed of light (17) and (18) there is a difference that can be related to the difference between pV_m constant and the speed of light. As a consequence we may say that between elementary physical entity and pV_m constant there is a direct relationship.

Having in view the general relation on gasses, the absolute temperature *K* is a key element. In table *2*, row *9* it is shown that the difference between normalized value for speed of light $\overline{x}_c$ and the normalized value for absolute temperature $\overline{x}_K$ is:

$$\overline{x}_c - \overline{x}_K = 1.000000-0.0300807(07) = 0.9699192(93). \quad (24)$$

If this value is divided 2^{190} it is obtained:

$$0.9699192(93)/2^{190} = 6.180682(32) \cdot 10^{-58}, \quad (25)$$

a value close to the elementary physical entity diameter that is *6.1803002130195…* " 10^{-2}.

In the final of this subchapter it is considered very important and relevant the following example concerning the electron mass.

The electron mass is equal to $9.10938291\cdot 10^{-31}$ *kg*.[11] If the mass of a kilogram is expressed as the mass of electrons is obtained the following normalized value associated with the mass:

$$1 \text{ kg} / 9.10938291\cdot 10^{-31} \text{ kg} = 1.09769201\cdot 10^{30} \quad (26)$$

If this result is divided by 2^{245} is obtained:

$$1.09769201\cdot 10^{30}/2^{245} = 1.941610467\cdot 10^{-44} \quad (27)$$

This result is very closed to the difference between the normalized value for the speed of light $\overline{x}_c$ and the normalized value for the Planck constant $\overline{x}_{\hbar^*}$ respectively $\overline{x}_c - \overline{x}_{\hbar^*} = 1.941598575\cdot 10^{-2}$ (table 2, row 3). If $1.941610467\cdot 10^{44}$ is divided by π *is* obtained $0.6180338069 \cdot 10^{-44}$ that is very close to $(\varphi\text{-}1) \cdot 10^{-44}$.

Conclusions

In the proposed numerical treatment approach, some fundamental physico-chemical constants can be re-written using a power-law model by taking number *2* as basis, thus a so-called "characteristic length" being highlighted every time.

If the "characteristic length" for a fundamental physico-chemical constant is related to the "characteristic length" of the light speed, every fundamental constant could be thus expressed in relative terms, irrespectively to the used unit system for the original values.

Calculations show that between the normalized values associated with the fundamental physico-chemical constants, the number *1* (or its decimal multiple and submultiple), the number π, the number φ or φ-*1* (where φ is the root of the equation $x^2 - x - 1 = 0$) and the power of the number *2* there are certain relationships. The powers of the number *2* are essential in such relationships.

These relationships could be used to explain in a non-conventional way, for instance some “liaisons” between subatomic mater structure. The article presents an example of how the mass of the proton could be obtained by a similarity analysis. At the same time there are presented examples concerning relationships between light and the electron mass or between light and the absolute temperature.

The relationships between the normalized values of some fundamental physico-chemical constants are similar with the relationships existing between elementary geometric figures (quadratic, rectangular, circular). In this context they could be presented in a unitary, coherent and synthetic way as in figure 1. The construction of figure 1 is based on the golden number φ (or golden section).

In the paper only relationships between light speed, Planck constant and pV_m constant are presented. But using dimensional analysis very interesting results concerning Avogadro constant NA,, electron charge e, even on the constant of gravitation Gk have been obtained. It is not goal of this paper to detail these results but it may be underlined that a figure like figure 1 could be used to present in a synthetic way correlations exiting between these fundamental constants.

Having in view these results, one can conclude that the proposed numerical analysis offers the possibility to established new and unexpected relationships between physico-chemical constants and between fundamental elements of the mater structure. At the same time, it is proved that when the relationships between physico-chemical phenomena are expressed using normalized (relative) values the great universal harmony might be emphasized.

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